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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2016

FIRST YEAR [BATCH 2016-19]

MATH FOR INDUSTRIAL CHEMISTRY [General]

Paper : I

Date : 19/12/2016

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A

Unit-I

(Answer any five questions) [5×5]

1. State De Moivre's theorem and use it to find the cube roots of –1. [2+3]

2. a) State the fundamental theorem of Classical Algebra. [1]
   b) State Descarte's rule of sign. [1]
   c) Let $f(x)=3x^8+x+1$. Determine whether there are odd number of real roots of $f(x)$ between x = 0 and x = 1 or not? [3]

3. If α,β,γ be the roots of the equation $x^3+px+q=0$, find an equation whose roots are $\frac{1}{\beta+\gamma},\frac{1}{\gamma+\alpha},\frac{1}{\alpha+\beta}$ and hence find $\sum\frac{1}{\beta+\gamma}$. [5]

4. Prove that [5]

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

5. Solve by Cardan's method $x^3-18x+35=0$. [5]

6. a) Define the eigenvalues and eigenvectors of a 2 x 2 real matrix. [1]
   b) Let $A=\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$ then find the eigenvalues of $\left(A^2+A^T\right)$. [4]

7. Show that the equations

$$\begin{aligned} x+\ \ y+\ \ z &= 6 \\ x+2y+3z &= 14 \\ x+4y+7z &= 30 \end{aligned}$$

are consistent and solve them. [5]

8. a) Find the characteristic equation of the matrix $A=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. [2]
   b) State Caley-Hamilton theorem. [1]
   c) Check Caley-Hamilton theorem for the matrix $A=\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$. [2]

Unit-II

(Answer <u>any five</u> questions) [5×5]

9. Consider the function, $f(x)=|x-1|+|x-2|^2$, $x\in\mathbb{R}$ ($\mathbb{R}$ is the set of all real numbers). Check whether $f(x)$ is continuous and differentiable at $x = 1$ and $x = 2$. [5]

10. Prove that if a function f is derivable at $x = a$ then f is continuous at $x = a$. Is the converse true? Justify. [3+2]

11. $$f(x)=\begin{cases} x & : \ 0<x<1 \\ 2-x & : \ 1\le x\le 2 \\ x-\frac{1}{2}x^2 & : \ x>2 \end{cases}$$

Show that f is continuous at $x = 1$ but not differentiable at $x = 1$. [5]

12. a) Evaluate $\underset{\substack{x\to 0\\ y\to 0}}{Lt}\, xy.\dfrac{x^2-y^2}{x^2+y^2}$. [3]

b) Check whether $f(x,y)=|x|+y$ is continuous at (0,1). [2]

13. a) State the Schwarz's theorem for a two variable function $f(x,y)$ at a point (a, b) in $D\ (D\subseteq\mathbb{R}\times\mathbb{R})$. [2]

b) If $u=f(x,y)$ be a homogeneous function of x and y of degree n having continuous partial derivatives, then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=nu$. [3]

14. If $v=2\cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\dfrac{\partial v}{\partial x}+y\dfrac{\partial v}{\partial y}+\cot\dfrac{v}{2}=0$. [5]

15 If $u=f(x,y)$ and $x=r\cos\theta,\ y=r\sin\theta$ then prove that $\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial u}{\partial y}\right)^2=\left(\dfrac{\partial u}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$. [5]

16. A function is defined by

$$f(x,y)=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$$
$$f(0,y)=f(x,0)=0$$

Show that $f_{xy}=f_{yx}$ at all points except (0,0). [5]

<u>Group – B</u>

(***Answer <u>any five</u> questions***) [5×5]

17. For the frequency table given below $\bar{X}=3\cdot 68$. Find the two missing frequencies [5]

Value of X	0	1	2	3	4	5	6	7	Total
Frequency	2	8	11	—	29	—	12	3	100

18. In a frequency table, the upper boundary of each class-interval has a constant ratio to the lower boundary. Show that the geometric mean (G) may be expressed as

$$\log G = A + \frac{k}{n}\sum_{i=1}^{r} f_i (i-1)$$

where $n = \sum_{i=1}^{r} f_i$, A is the logarithm of the class-mark of the first interval and k is the logarithm of the ratio between the upper and lower boundaries. [5]

19. The median and mode of the following frequency distribution are respectively 27 & 26. Find a and b. [5]

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	a	20	12	b

20. Define 'standard deviation' for a frequency distribution.

A student obtained the mean and standard deviation of 100 observations as 40·1 and 5·0 respectively. It was later found that he copied one value 50 wrongly instead of the correct value 40. Find the correct mean and correct standard deviation. [1+4]

21. Prove the following statements.

a) 'Standard deviation' is the least 'root-mean-square deviation'.

b) The 'standard deviation' cannot be smaller than 'mean deviation about mean'. [2+3]

22. r_{xy} is the correlation coefficient between x & y. Define $u = \frac{x-a}{c}$ & $v = \frac{y-b}{d}$. Find r_{uv}. [5]

23. If $u = x\cos\theta + y\sin\theta$, $v = y\cos\theta - x\sin\theta$ and the variables u and v are uncorrelated, then prove that $\tan 2\theta = \frac{2r_{xy}s_x s_y}{s_x^2 - s_y^2}$ where all the symbols have their usual significance. [5]

24. You are given the following data :

Variable	X	Y
Mean	47	96
Variance	64	81

Correlation coefficient between X and Y is 0·36.

Determine the equations of regression lines. Calculate Y when X = 50, and X when Y = 88. [4+1]

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